PhD thesis proposal– 2026

- **Title: PDE and metric aspects of nonlocal operators.**

- **Research units:**
  IRMAR, UMR-6625, INSA Rennes, Univ Rennes, France.
  Vador E105-04, TU Wien, Austria.

- **Theme:** PDE, Metric analysis.

- **Key words:** Nonlinear partial differential equations, Hamilton-Jacobi equations,

- **Names, emails, institutions of doctoral advisors.**

  Olivier Ley, Univ Rennes, INSA Rennes, IRMAR, France.
  olivier.ley@insa-rennes.fr
  https://ley.perso.math.cnrs.fr/

  Aris Daniilidis, TU Wien, Austria.
  aris.daniilidis@tuwien.ac.at
  https://www.arisdaniilidis.at/

- **Name, emails, institutions of doctoral coadvisors**
  Sebatián Tapia García, TU Wien, Austria.
  sebastian.tapia.garcia@tuwien.ac.at
  https://sites.google.com/view/sebastian-tapia-garcia/home

PhD Research proposal

This PhD project centers on the study of Hamilton–Jacobi–Bellman equations, with particular emphasis on the theory of viscosity solutions and infinite-horizon control problems. It also investigates several components of weak KAM theory, including its extension to general metric spaces, as well as abstract notions of descent such as the De Giorgi slope, global slope, and various notions of average slope. A central aim of the project is to develop and clarify the connections between these different frameworks.

The proposed research concerns the investigation of appropriate notions of viscosity solutions for abstract descent moduli, with special focus on existence, uniqueness and stability of solutions. This corresponds to the starting point of the thesis. Further, and depending on the progresses of the PhD candidate, there are two proposed lines of research. First, due to the fact that the notion of viscosity solution was developed to solve Hamilton Jacobi equations, the link between descent moduli and Hamilton-Jacobi equations defined in general metric spaces can be further investigated. Second, connections between the (nonlocal) fractional Laplace equation and global slopes has been investigated in [2]. It is interesting to study asymmetric versions of the above operator from

both, a PDE and a purely metric viewpoints. Some connections with a nonlocal Tug-of-war game could be also addressed.

The main tools to be developed in this project are the ones of viscosity theory to tackle problems from a PDE point of view, and also the ones of variational analysis to deal with problems in the metric case.

This research project is part of the French-Austrian International Collaborative Research Project ANR SABOCPR / FWF PIN 4368225. The 3-year doctoral contract will be carried out at the Université de Rennes, France. It is expected that the PhD student spends the half of his/her PhD journey at the Université de Rennes and the other half at the TU Wien, Austria.

References

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