Master Internship

High-order structure-preserving methods for hyperbolic PDEs on curved domains with Cartesian grids

Laboratoire Jacques-Louis Lions, Université Paris Cité
Paris, France • 5 months • Flexible starting date

Context

Numerical simulations of hyperbolic systems of conservation laws on complex geometries play a central role in many applications, ranging from compressible gas dynamics around airfoils to shallow-water flows in urban environments. High-fidelity simulations require accurate discretizations of both the governing equations and the boundary conditions, especially when curved or moving boundaries are involved. Classical body-fitted approaches simplify the enforcement of boundary conditions but become prohibitively expensive when dealing with geometrically complex domains, where curvilinear mesh generation is a well-known bottleneck.

Embedded (or unfitted) boundary methods overcome this limitation by discretizing the physical domain on a Cartesian mesh that does not conform to the boundary. Boundary conditions are then imposed through a suitable modification of the discretization at interface elements, avoiding costly remeshing while retaining geometric flexibility. However, this flexibility comes at the price of a geometric error that can spoil the global accuracy, particularly when high-order methods are employed.

At the same time, standard discretizations of multi-dimensional conservation laws rely on stabilization techniques that introduce numerical diffusion along the direction normal to each interface. This diffusion destroys genuinely multi-dimensional stationary states, such as divergence-free flows, which are instead preserved by stationarity-preserving (or structure-preserving) methods. These methods modify the stabilization term so that it vanishes at stationary states, allowing the discrete scheme to preserve them exactly.

This internship is part of the **HOPE project** (High Order structure Preserving Embedded methods for nonlinear conservation laws), which aims to unify high-order embedded boundary methods with stationarity-preserving discretizations in a single computational framework, capable of delivering high accuracy on coarse, non-conforming meshes.

Description

The goal of this internship is to design, analyze, and implement a high-order boundary treatment for hyperbolic systems of conservation laws discretized with structure-preserving methods on embedded boundaries.

The intern will work within the HOPE framework and will contribute to the following activities:

- Review of existing boundary treatments and high-order structure-preserving schemes, with particular attention to the stationary-preserving global-flux formulation in the finite element setting.
- Design of a high-order embedded boundary condition imposition compatible with the stationarity-preserving discretization
- Numerical implementation of the method in a research code, with tests on manufactured solutions and vortex-like flows to assess high-order accuracy and the preservation of stationary states.

- Comparison with standard approaches to quantify the gains in accuracy and robustness.

Required Skills

- **Background:** Last year of Master's Degree in applied mathematics, engineering, scientific computing, or related fields.
- **Technical skills:** Finite Element Method, numerical methods for PDEs, programming in one or more languages (Python, Fortran, Matlab, C/C++)
- **Language:** Languages of communication can be English, French or Italian.

Additional information

This position is for a **5-month** internship, with a flexible starting date.

The internship will be supervised by **Mirco Ciallella** and will be carried out at the **Laboratoire Jacques-Louis Lions** (Université Paris Cité) situated in Paris, France (Place Aurélie Nemours, 75013). The project will be done in collaboration with **Davide Torlo** (Università di Roma La Sapienza, Italy) and **Mario Ricchiuto** (INRIA Bordeaux, France)

The internship reward is about 540 euros per month (plus partial refund of transportation costs).

Contact

Send your application (CV, transcript, short motivation letter) to the contact person below.

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References

1. Ciallella, M. (2026). Minimization-based embedded boundary methods as polynomial corrections: a stability study of discontinuous Galerkin for hyperbolic equations. *Journal of Scientific Computing*. Link to the article.
2. Barsukow, W., Ricchiuto, M., Torlo, D. (2025). Structure preserving nodal continuous finite elements via global flux quadrature. *Numerical Methods for Partial Differential Equations*. Link to the article
3. Barsukow, W., Ciallella, M., Ricchiuto, M., Torlo, D. (2026). Genuinely multi-dimensional stationarity preserving Finite Volume formulation for nonlinear hyperbolic PDEs. *Journal of Computational Physics*. Link to the article