Master Internship

Stability characterization of explicit-implicit high-order embedded boundary methods for linear hyperbolic systems

Laboratoire Jacques-Louis Lions, Université Paris Cité
Paris, France • 5 months • Flexible starting date

Context

Numerical simulations of hyperbolic systems of conservation laws on complex geometries play a central role in many applications, ranging from compressible gas dynamics around airfoils to flows inside rocket nozzles. High-fidelity simulations require accurate discretizations of both the governing equations and the boundary conditions, especially when curved or moving boundaries are involved. Classical body-fitted approaches simplify the enforcement of boundary conditions but become prohibitively expensive when dealing with geometrically complex domains, where curvilinear mesh generation is a well-known bottleneck.

Embedded (or unfitted) boundary methods overcome this limitation by discretizing the physical domain on a Cartesian mesh that does not conform to the boundary. Boundary conditions are then imposed through a suitable modification of the discretization at interface elements, avoiding costly remeshing while retaining geometric flexibility. However, this flexibility comes at the price of a geometric error that can spoil the global accuracy and, more critically, trigger numerical instabilities, particularly when high-order methods are employed for hyperbolic problems. Understanding and controlling these instabilities is therefore essential to enable reliable high-fidelity simulations on complex, possibly moving, geometries.

This internship is part of the **PHOENICS project** (Provably-stable High Order Embedded methods for Nonlinear Conservation lawS), which aims to establish a rigorous stability framework for high-order embedded boundary treatments, integrate them within adaptive and parallel infrastructures, and validate them on realistic applications.

Description

The goal of this internship is to characterize the stability of high-order embedded boundary methods within an explicit/implicit time integration framework, and to analyze the impact of boundary conditions on the global stability of the discretization.

The intern will work within the PHOENICS framework and will contribute to the following activities:

- Review of existing embedded boundary treatments and their stability properties, with particular attention to the Shifted Boundary method (SBM) in the Discontinuous Galerkin framework.
- Analysis of linear hyperbolic systems to study how boundary conditions, and their interplay with the incoming and outgoing characteristic variables, affect the stability of the scheme.
- Stability characterization comparing standard explicit and novel high-order explicit-implicit time integration strategies.
- Implementation of the stability analysis in a numerical code and validation on benchmark problems.

Required Skills

- **Background:** Last year of Master's Degree in applied mathematics, engineering, scientific computing, or related fields.
- **Technical skills:** Numerical methods for PDEs (Finite Volume, Finite Element), programming in one or more languages (Fortran, Python, Matlab, C/C++)
- **Language:** Languages of communication can be English, French or Italian.

Additional information

This position is for a **5-month** internship, with a flexible starting date.

The internship will be supervised by **Mirco Ciallella** and will be carried out at the **Laboratoire Jacques-Louis Lions** (Université Paris Cité) situated in Paris, France (Place Aurélie Nemours, 75013).

The internship reward is about 540 euros per month (plus partial refund of transportation costs).

Contact

Send your application (CV, transcript, short motivation letter) to the contact person below.

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References

1. Ciallella, M., Gaburro, E., Lorini, M., Ricchiuto, M. (2023). Shifted boundary polynomial corrections for compressible flows: high order on curved domains using linear meshes. *Applied Mathematics and Computation*. Link to the article.
2. Ciallella, M. (2027). Stability analysis of discontinuous Galerkin with a high order embedded boundary treatment for hyperbolic equations. *Applied Mathematics and Computation*. Link to the article.
3. Ciallella, M. (2026). Minimization-based embedded boundary methods as polynomial corrections: a stability study of discontinuous Galerkin for hyperbolic equations. *Journal of Scientific Computing*. Link to the article.