

Phd Thesis in Applied Mathematics (2026-2029): An impulsive approach to optimal control of cellular resource allocation*

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1 Biological context

Predicting microbial growth rates is of great importance, in order to improve our comprehension of actual ecosystems and to understand how they respond to global change and anthropic pressure. A valuable approach to studying microbial growth is the examination of resource allocation [7]. Given that microorganisms have likely evolved strategies to maximize their growth, metabolism can be modeled as an optimization problem. In models of resource allocation, proteins must be distributed across various cellular functions (substrate uptake, protein synthesis, etc.) in order to maximize growth rate. This approach thus motivates a dynamical perspective on microbial growth and resource allocation leading to the formulation of optimal control problems (OCPs) such as in [6]. However, a crucial aspect has so far been overlooked in studies on resource allocation: *cell division*, which is precisely the focus of the present thesis. Therefore, we aim to integrate this division phenomenon into the previously studied models providing a significantly more realistic description of the microorganism dynamics. This integration will necessarily introduce certain complexities and will require a suitable methodological and numerical analysis.

2 Scientific objectives and organization

The main objective will be to understand, within resource-allocation models, the alternation between microorganism growth and cell division. Cell division naturally leads to the introduction of jumps in the dynamics (when a cell divides), and the problems previously studied now need to be addressed within an *impulsive optimal control framework*, which constitutes the novelty of this project. As a consequence, the state of the system becomes discontinuous in time. In addition, jumps occur on prescribed manifolds whenever a concentration reaches a specified threshold. Once an OCP, in the spirit of the one studied in [6], is formalized within the impulsive framework, the objective will be to determine both the optimal control laws for the resource-allocation model and the optimal switching times corresponding to division. While theoretical optimality conditions for impulsional optimal control problems are well established [1, 4]¹, numerical methods are more challenging due to the state's discontinuities. So, the project will set up an appropriate impulsive framework to analyze optimal solutions, notably through Pontryagin-type conditions. This framework will serve as a basis for developing new approximation techniques to approximate the solutions of the impulsive optimal control problems under consideration (drawing, for example, on the approaches in [2, 3]). The organization of the thesis will therefore be structured around both biological modeling aspects and a methodological component, with the aim of developing simulation methods for the optimal solutions:

- **1. Modeling of allocation resource problems including cell division.** The aim will be to formalize an OCP for a core resource allocation model that includes cell division, and then to solve it in order to

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¹Seminal work on impulsive systems was initiated by Bensoussan and Lions in 1982.

determine the number and timing of divisions and to analyze their effects on cell growth. We will be interested in observing how *turnpike optimal strategies* are affected by discontinuities in this context.

- **2. Optimality conditions for impulsive optimal control problems.** Parallel to point 1, we plan to derive efficient ways for the synthesis of optimal controls in the context of impulsive optimal control problems including a switching manifold. The aim will be twofold. First, we will be interested in revisiting necessary optimality conditions for impulsive problems and then, to extend them to this setting. Additionally, we shall complete this approach by developing numerical schemes and regularization techniques in this setting.
- **3. Extension to other biological models.** Finally, we will also seek to extend the core OCP to address more specific biological questions, such as how growth and cell division coordinate with day-night cycles [6] or interact with nutrient-quota dynamics [5] in microalgae and other microorganisms.

3 Grant and required profile

A fellowship from Ecole Doctorale d'Avignon (ED 536 agrosociences, Avignon University) will be dedicated to LMA (Laboratoire de Mathématiques d'Avignon) for 2026. It will cover three years of PhD (01/10/2026 to 31/09/2029) and the usual salary is about 1650 euros net per month. There is also a possibility of doing teaching duties as *allocataire moniteur* during the three years (no more than 64h per year which is the maximum allowed). Candidates should have a Master degree in applied mathematics or in mathematics. Candidates currently in Master's 2 degree are also eligible to apply. A solid background in Applied Mathematics (mastering the entire Bachelor's degree is essential) is required. Candidates should have a strong interest in biological problems, as well as the ability to solve and implement solutions for challenges derived from modeling, in addition to the necessary theoretical skills. Candidates should also have a background in the following fields: **optimal control of ODEs or PDEs ; optimization and numerical analysis ; dynamical systems and ordinary differential equations.** Programming skills for implementing optimization methods solving concrete optimal control problems will also be appreciated but are not mandatory.

4 How to apply?

Interested students should send their CV, references, and a letter of motivation to T erence Bayen, Francis Mairet and Walid Djema (e-mail address in footnote²). Candidates that are interested to apply to the thesis project will also have to apply on the ADUM web application.

Dead-line : around may, 15th. After a preselection of candidates by advisors and the director for mathematics, those selected ones will do an audition (interview) organized by the Ecole doctorale d'Avignon. The admission jury of the Ecole Doctorale will then rank the shortlisted candidates.

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