Invertible Neural Networks for the backstepping method

M2 Internship proposal

Practical informations

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- Internship duration: 5 to 6 months with funding.
- Projected start of the internship : beginning of April.
- PhD funding possibility: ongoing application through ENACT AI cluster.

Prerequisites

- Be a M2 student.
- Strong background in PDEs and in AI. Having taken a control course will be an asset.

Internship description

The internship consists in investigating Invertible Neural Networks (INN) architectures for the construction of feedbacks of stabilizable finite dimensional systems. More specifically, we are interested in the backstepping approach. Assume $(A, B) \in M_n(\mathbb{R}) \times \mathbb{R}^n$ and assume

$$\begin{cases} x'(t) = Ax(t) + Bu(t), \quad t \in (0, T_f) \\ x(0) = x_0 \in \mathbb{R}^n \end{cases} \tag{1}$$

is controllable with $T_f > 0$ and $u_0 \in L^2(0, T_f)$. The controllability implies the rapid stabilization of the closed-loop system

$$\begin{cases} x'(t) = (A + BK)x(t), \quad t \in (0, T_f) \\ x(0) = x_0 \in \mathbb{R}^n \end{cases} \tag{2}$$

that is, for any $\lambda > 0$, there exists a feedback $u(t) = Kx(., t)$ such that solution $x(t)$ to (2)

$$\|x(t)\| \leqslant Ce^{-\lambda t}\|x_0\|, \quad t \in (0, T_f).$$

Many constructions of such feedbacks K exist in the literature. We are interested here in the backstepping approach, that is, to find an invertible transformation T such that $y(t) = Tx(t)$, where y is the solution of the target stable system

$$\begin{cases} y'(t) = (A - \mu I)y(t), \quad t \in (0, T_f) \\ y(0) = y_0 \in \mathbb{R}^n \end{cases} \tag{3}$$

The target system (3) is exponentially stable at rate λ if $\mu = \lambda + \max\{\lambda_k\}$ where $\{\lambda_k\}_{k=1}^n$ is the set of eigenvalues of A. The invertibility of T yields the exponential stability of (1) by choosing $y_0 = Tx_0$

$$\|x(t)\| \leqslant \|T^{-1}y(t)\| \leqslant e^{-\lambda t}\|T^{-1}\|\|y_0\| \leqslant Ce^{-\lambda t}\|x_0\|.$$

The practical computation of T and K might prove to be tedious, especially in the case where (2) is a PDE.

In this internship, we are aiming at investigating different AI approaches to construct T_σ, an approximation of T. More specifically, we will be interested in approaches extendable to the PDE framework. The philosophy we want to use is that T is suppose to learn the dynamic between $x(t)$ and $y(t)$. We also want to ensure that T_σ is invertible. A natural extension of the internship is a PhD thesis on the extension for the backstepping method applied to PDEs, with applications to the reconstruction of solidification process from experimental datas.

References

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[2] Jean-Michel Coron. Stabilization of control systems and nonlinearities. In *Proceedings of the 8th International Congress on Industrial and Applied Mathematics*, pages 17–40. Higher Ed. Press, Beijing, 2015.

[3] Ludovick Gagnon, Amaury Hayat, Shengquan Xiang, and Christophe Zhang. Fredholm transformation on laplacian and rapid stabilization for the heat equation. *Journal of Functional Analysis*, 283(12):109664, 2022.